PHEV Energy Use Estimation: Validating the Gamma Distribution for Representing the Random Daily Driving Distance

Zhenhong Lin (linz@ornl.gov), Jing Dong (dongj@ornl.gov), Changzheng Liu (liuc2@ornl.gov), David L. Greene (digreene@ornl.gov)
Oak Ridge National Laboratory, National Transportation Research Center, 2360 Cherahala Boulevard, Knoxville, Tennessee 37932

Abstract

***Key Message*: The Gamma distribution is a reliable assumption for the random daily vehicle miles traveled (DVMT) in the context of plug-in hybrid electric vehicle (PHEV) energy analysis.**

- **The Gamma assumption is validated using continuous GPS travel data of 382 vehicles, collected in the Seattle metropolitan area over an 18-month period.**
- **The conclusion is drawn based on practical significance of the errors in predicting PHEV energy use and cost, not necessarily based on statistical significance.**

***Significance of the finding*:**

Gamma distribution assumption for random DVMT can be used in a variety of applications, such as developing vehicle consumer choice models, quantifying range anxiety for battery electric vehicles, investigating the role of charging infrastructure, and constructing online calculators that provide personal estimates of PHEV energy use.

Background

Plug-in hybrid electric vehicles (PHEV)

- **operate on both electricity and fuel**
- **reduce petroleum use and pollutant emissions**
- **avoid the range anxiety associated with all-electric vehicles and the infrastructure rollout challenge faced by fuel cell vehicles**

Variation in Daily Vehicle Miles Traveled (DVMT)

- **Ignoring the DVMT variation would underestimate fuel use and overestimate electricity use of PHEVs**
- **National longitudinal data --- ideal but costly**
- **Multi-vehicle single-day data --- improvement, but not enough**
- **Approximation method is needed**

Data

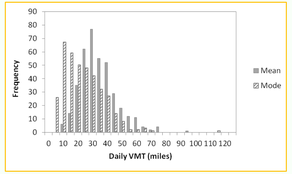
GPS tracking data

- **Collected in the Seattle metropolitan area**
- **Over an 18-month period (from November 2004 to April 2006)**
- **Daily travel distances of 382 volunteer drivers**

Why Gamma?

Describing the data

- **non-negative**
- **flexible skewness**
- **most drivers drive 20~40 miles on average**
- **the frequent trips are in the 10~30 miles range**



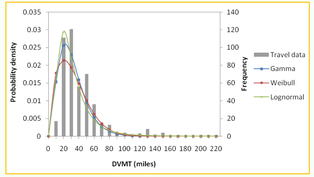
Easy to estimate parameters

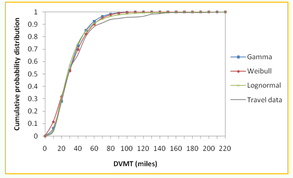
	Probability density function	Parameters	Mean	Mode
Gamma	$p(x)=\frac{1}{\theta^k\cdot\Gamma(k)}x^{k-1}\cdot e^{-x/\theta}$	$k>0$ – shape $\theta>0$ – scale	$k\theta$	$(k-1)\theta$, for $k\geq 1$
Weibull	$p(x)=\frac{k}{\theta}\cdot(\frac{x}{\theta})^{k-1}\cdot e^{-(x/\theta)^k}$	$k>0$ – shape $\theta>0$ – scale	$\theta\cdot\Gamma(1+\frac{1}{k})$	$\theta(\frac{k-1}{k})^{1/k}$, for $k>1$
Lognormal	$p(x)=\frac{1}{x\cdot\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$	$\sigma^2>0$ – shape $\mu\in R$ – log scale	$e^{\mu+\sigma^2/2}$	$e^{\mu-\sigma^2}$

- **MLE of the probability distribution parameters for each driver**
- **Rank the distributions in order of decreasing log-likelihood values**

Table. Comparison of Three Alternative Distributions

Rank	Gamma	Weibull	Lognormal
1	164	127	91
2	202	158	22
3	16	97	269





Easy for mathematical manipulation

The Bohr–Mollerup theorem allows derivation of the formula of partial expectation based on the Gamma distribution, which allows faster closed-form calculation in the PHEV energy use estimation.

Methodology

PHEV Energy Use

- **Gasoline-based PHEVs are assumed for the validation.**
- **The gasoline and electricity consumptions by a PHEV are determined by dynamometer-based performances, powertrain control algorithms, charging patterns, and driving patterns.**

$$G_{PHEV,gamma,i}=\frac{1}{x_{avg}}\left[\begin{array}{l}DynoG_{cd}\cdot\int_0^{R_{cd}}x\cdot p_i(x;k,\theta)\cdot dx+DynoG_{cs}\cdot\int_{R_{cd}}^{+\infty}x\cdot p_i(x;k,\theta)\cdot dx\\+(DynoG_{cd}-DynoG_{cs})\cdot R_{cd}\cdot(1-\int_0^{R_{cd}}p_i(x;k,\theta)\cdot dx)\end{array}\right]$$

$$E_{PHEV,gamma,i}=\frac{DynoE_{cd}}{x_{avg}}\left[\int_0^{R_{cd}}x\cdot p_i(x;k,\theta)\cdot dx+R_{cd}\cdot(1-\int_0^{R_{cd}}p_i(x;k,\theta)\cdot dx)\right]$$

$$x_{avg}=\int_0^{+\infty}x\cdot p_i(x;k,\theta)\cdot dx$$

Table. Assumptions of PHEV Dynamometer-based Performances

	PHEV10	PHEV20	PHEV30	PHEV40
CD range (mile)	10	20	30	40
CS gasoline rate (gallon/mile)	0.0237	0.0238	0.0241	0.0244
CD gasoline rate (gallon/mile)	0.0095	0.0082	0.0070	0.0057
CD electricity rate (Wh/mile)	180.0	197.5	218.5	238.6

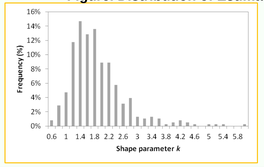
Maximum Likelihood Estimation (MLE)

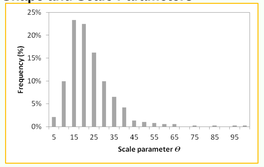
DVMT of each driver is viewed as a random variable following a Gamma distribution. The shape and scale parameters, charactering the DVMT distribution of a driver, are estimated using the GPS tracking data and MLE.

$$L(k,\theta;x)=p(x_1,\cdots,x_n;k,\theta)=\prod_{i=1}^{n}p(x_i;k,\theta).$$

- **382 sets of shape and scale parameters are estimated for 382 drivers**
- **Both parameters vary significantly among the participant drivers**

Figure. Distribution of Estimated Shape and Scale Parameters





Results and Discussion

The Gamma distribution for the random DVMT in PHEV energy analysis is deemed as a valid assumption if the resulting relative errors in estimation of oil saving, grid impact, and private energy cost reduction are small.

$$\delta_{C,i}=(\Delta C_{gamma,i}/\Delta C_{data,i})-1$$
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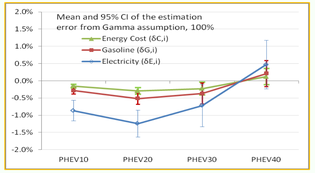

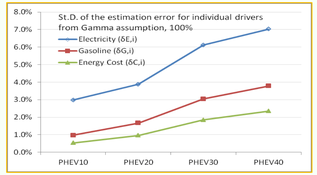

***Fleet-wide*: Errors (Sample mean and standard error)**

- **Oil savings: range from -0.7% to -0.3% for PHEV20s and even smaller (absolute value) for other PHEVs.**
- **Grid impact: range from -1.7% to -0.7% for PHEV20s and smaller (absolute value) for other PHEVs.**

***Individual*: Energy cost errors**

- **standard deviations range from 0.5% for PHEV10s to 2.3% for PHEV40s.**

Conclusions

Summary and limitations

This study validates the Gamma assumption for the random DVMT in the context of PHEV energy analysis.

- **The validation is based on practical significance of the errors in predicting PHEV energy use and cost.**
- **The validity conclusion is drawn based on travel data collected for the Seattle region.**
- **The validity conclusion may not be applicable for general purposes beyond PHEV energy analysis.**

Implications

- **The Gamma distribution can be easily specified with very few pieces of driver information and is relatively easy for mathematical manipulation.**
- **The finding that the Gamma distribution is valid and reliable paves the way for the Gamma distribution to be assumed for analyzing energy use of PHEVs in the real world.**

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